

Material: Sections 4.1, 4.2, 4.3, 4.4, 4.6, 4.7, 7.1, 7.2, 7.3, 7.4.

Duration: 1h.30

Exercise 1. (15 points)

Find the general solution of $y'' - y' - 2y = 5x^2$.

Hint. Find a particular solution which is a polynomial.

Exercise 2. (15 points)

Solve the differential equation $y'' + 2y' - 3y = \sin(2x)$.

Exercise 3. (10 points)

Find the general solution of

$$x^2y'' + xy' - y = 0 \quad \text{for } x > 0.$$

Exercise 4. (15 points)

Use the variation of parameters method to solve the equation

$$y'' - 2y' + y = \frac{e^x}{x} \quad \text{for } x > 0.$$

Exercise 5. (25 points)

a) Find a function whose Laplace transform is $\frac{1}{(s^2 + 4)^2}$.

b) Solve the initial value problem

$$\begin{aligned} y'' + 4y &= \sin(2t) \\ y(0) &= 0, y'(0) = 1. \end{aligned}$$

Do you see any physical meaning of this problem?

c) (*Bonus*) Find a function whose Laplace transform is $\frac{1}{(s^2 + 4)^3}$.

Exercise 6. (20 points)

Solve by two different methods the initial value problem

$$\begin{aligned} y'' - y' + y &= e^{-t} \\ y(0) &= y'(0) = 0. \end{aligned}$$